

Fermat's Principle

Pierre de Fermat (1650)

Fermat's principle is an important principle in optics which explains the propagation of light.

OPTICAL PATH (Δ)

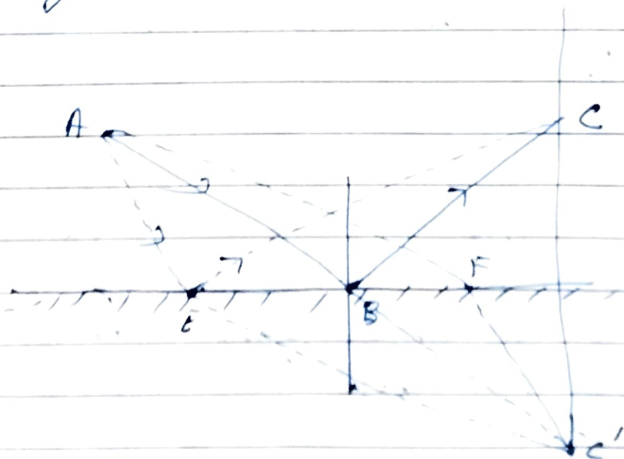
Geometrical path \rightarrow

The geometrical path between two points is the actual distance travelled by light between two points.

\Rightarrow optical path is defined as the product of the geometrical path and the refractive index of the medium.

$$\Delta = n \cdot r$$

Before Fermat's law it is assumed that the light travels from one point to another point along the shortest path which can be explained with the help of Reflection.



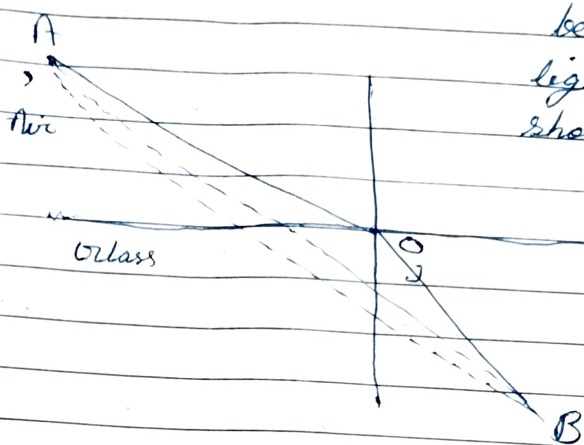
$$AB + BC = AB + BC' = AC'$$

$$AE + EC = AE + EC' > AC'$$

In case of

Reflection It is clear that light propagate through the shortest path.

But in case of refraction light does not follow the shortest path.



between P to Q
light follow
shortest path

$$\frac{dt}{ds} = 0$$

Fermat suggest that principle of shortest path can be replaced by principle of shortest time or "least time".

Light travels from one to another point along that path which requires least time.



In dt time light travel a distance of dL then

$$dt = \frac{dL}{v}$$

Total time

$$t = \int_p^Q dL$$

$$\left[\mu = \frac{c}{v} \right]$$

$$\Rightarrow t = \int_p^Q \frac{\mu dL}{c} = 0$$

$$= \mu \int_p^Q dL = \Delta$$

$$\Rightarrow \boxed{t = \Delta/c}$$

$$t = \frac{\Delta}{c}$$

$$\therefore t \propto \Delta$$

Since time traverse is proportional to optical path length Δ

Hence we can say that

light travels along a path having the minimum optical path length

$$\boxed{\frac{d\Delta}{dS} = 0}$$